PLASMA DENSITY AND ELECTRIC FIELD STRENGTH DISTRIBUTIONS AT A BOUNDARY WITH AN ELECTRODE

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Close to the cathode in an arc discharge, or to the surface of a probe operating on the ionic branch of its characteristic, conditions are obtained under which the Langmuir layer freely passes ions coming from the plasma, while the reverse ion flow is virtually zero. The plasma density, ion distribution, and electric field-strength close to the electrode are found in the present paper. The extrapolated length is evaluated for the plasma density. The absolute value of the electric field strength increases logarithmically at the boundary with the electrode.

1. The expression

$$f(u, 0) = \sqrt{2/\pi}\, n(0) \exp(-u^2)\, \Theta(u) \,/\, X(-u) \tag{1.1}$$

was obtained in [1] for the ion distribution at the boundary with an electrode, under the condition that the potential drop in the Langmuir layer substantially exceeds the mean thermal energy of an ion or electron and is directed in such a way that electrons leaving the plasma are directed backwards.

Such conditions occur near the cathode in an arc discharge, since the cathode drop is substantially greater than the mean thermal energy even in the case of a low-voltage arc. Similar conditions obtain around a probe operating under the ionic current condition:

$$X(-u) = \frac{\exp \Gamma(-u)}{u}, \quad \Gamma(-u) = -\int_0^\infty \left[1 - \frac{1}{\pi} \operatorname{Arc\,tg} \frac{\sqrt{\pi} t \exp(-t^2)}{\lambda(t)}\right] \frac{dt}{t+u}$$
$$\lambda(t) = 1 + \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-y^2)}{y-t} dy, \quad \Theta(u) = \begin{cases} 1 & (u > 0) \\ 0 & (u < 0) \end{cases} \tag{1.2}$$

where $f(u, \xi)$ is the ion distribution function, normalized to the plasma density $n(\xi)$, u is the dimensionless velocity, and ξ the dimensionless coordinate:

$$u = -v/v_0, \quad \xi = x/(v_0 \tau), \quad v_0 = (2T/M)^{1/2}$$

where x is a coordinate orthogonal to the electrode, v is the ion velocity component along the x axis, τ is the ion distribution function relaxation time, which will be assumed constant, T is the atom temperature, and M is the mass of the ions.

The paper investigates the plasma density, the ion distribution, and the electric field (within the Langmuir layer) as functions of the distance from the electrode.

2. We introduce the auxiliary function

$$\psi(u, \xi) = f(u, \xi) + \alpha \pi^{-1/2} \exp(-u^2)\, n(\xi) \qquad (\alpha = T_e/T)$$

which is a solution of the equation [1]

$$-u d\psi / d\xi + \psi = (1 + \alpha)\, \pi^{-1/2} \exp(-u^2)\, n(\xi)$$

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where T_e is the electron temperature. Denoting by $\Psi(u, k)$ the Fourier transform of $f(u, \xi)$, we have [1]

$$\Psi(u, k) = \frac{1}{1+iku}\left(\frac{\exp(-u^2)\,N(k)}{\sqrt{\pi}} - \frac{u\psi(u, 0)}{\sqrt{2\pi}}\right) \tag{2.1}$$

The function N(k) is equal to the Fourier transform of $n(\xi)$ multiplied by $1 + \alpha$, and may be found as follows from (2.1). Since $n(\xi)$ and $f(u, \xi)$ have an algebraic growth [1] as $\xi \to \infty$, $\Psi(u, k)$ can have no singularities when Im $k > 0$. Hence,

$$\exp(-u^2)\,N(k) = u\psi(u, 0)\,/\,\sqrt{2} \qquad (k = i\,/\,u,\ u > 0)$$

Using this and (1.1), the expression

$$N(k) = \frac{in(0)}{k\sqrt{\pi}}\left(\frac{1}{X(-i/k)} + \frac{\alpha}{\sqrt{2}}\right) \tag{2.2}$$

holds for all values of k. Hence, instead of (2.1),

$$\Psi(u, k) = \frac{1}{1+iku}\left[\frac{in(0)\exp(-u^2)}{k\sqrt{\pi}}\left(\frac{1}{X(-i/k)} + \frac{\alpha}{\sqrt{2}}\right) - \frac{u\psi(u, 0)}{\sqrt{2\pi}}\right]$$

3. It is clear from (1.2) that the function $X(-i/k)$ appearing in $\Psi(u, k)$ is analytic everywhere except on the cut $(0, -i\infty)$. Its limiting values X_+, X_- on the edges of the cut satisfy [1]

$$X_+(t)\,/\,X_-(t) = \Lambda_+(t)\,/\,\Lambda_-(t), \qquad \Lambda_\pm(t) = \lambda(t) \pm i\sqrt{\pi}t\exp(-t^2), \qquad X_+(t)\,X(-t) = 2\Lambda_+(t), \qquad t > 0$$

Hence, $X(-i/k)$ vanishes only when $k = 0$, near which it has the expansion [1]

$$X(-i/k) = -ik + k^2 l_0 + O(k^3) \qquad (l_0 = 1.016)]$$

Knowing the analytic properties of $X(-i/k)$, the required functions $n(\xi)$ and $f(u, \xi)$ can be found by performing the inverse Fourier transformation. The result is

$$n(\xi) = \frac{\sqrt{2}n(0)}{1+\alpha}\left(\xi + l_0 + \frac{\alpha}{\sqrt{2}} - \frac{1}{2\sqrt{\pi}}\int_0^\infty \frac{dt\exp(-\xi/t - t^2)\,X(-t)}{\Lambda_+(t)\,\Lambda_-(t)}\right) \tag{3.1}$$

$$f(u, \xi) = n(0)\exp(-u^2)\sqrt{\frac{2}{\pi}}\left(u + \frac{\xi + l_0 + \alpha/\sqrt{2\pi}}{1+\alpha} - \Theta(-u)\frac{\exp(\xi/u)\,\lambda(u)\,X(u)}{2\Lambda_+(u)\,\Lambda_-(u)}\right. \tag{3.2}$$

$$\left. + \frac{u}{2\sqrt{\pi}}\int_0^\infty \frac{dt\exp(-\xi/t - t^2)\,X(-t)}{(t+u)\,\Lambda_+(t)\,\Lambda_-(t)} - \frac{1}{2\sqrt{\pi}(1+\alpha)}\int_0^\infty \frac{dt\exp(-\xi/t - t^2)\,X(-t)}{\Lambda_+(t)\,\Lambda_-(t)}\right)$$

4. This result will be analyzed. From (3.2), as $\xi \to \infty\ (x \gg \nu_0\tau)$, the distribution function clearly tends to the asymptotic form

$$f(u, \xi) \simeq n(0)\exp(-u^2)\sqrt{\frac{2}{\pi}}\left(u + \frac{\xi + l_0 + \alpha/\sqrt{2}}{1+\alpha}\right) \tag{4.1}$$

This expression is the usual diffusion function; with ξ large, the term proportional to the velocity becomes small compared with the symmetric part of the distribution function. In the absence of a field, α has to be put equal to zero [2].

When $\xi \gg 1$, the density is a linear function of the distance:

$$n(\xi) = n(0)\sqrt{2}(\xi + l_0 + \alpha/\sqrt{2})\,/\,(1+\alpha) \tag{4.2}$$

From this last expression, the extrapolated length depends on the electron temperature and is equal to

$$l_0 + T_e/(\sqrt{2}T)$$

The rate of relaxation to the linear law is determined by the last term in the parentheses in (3.1). The graph of this function is given in Fig. 1; it can be seen to decrease rapidly with the distance; even when $x = v_0\tau$, its contribution to (3.1) is less than 3% of the remaining terms.

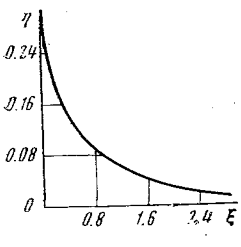

Fig. 1

Since the electrons have an equilibrium distribution, the plasma density is connected with the field by

$$E = -\frac{T_e}{en\tau v_0}\frac{dn}{d\xi}, \qquad \frac{dn}{d\xi} = \frac{n(0)\sqrt{2}}{1+\alpha}\left(1+\frac{1}{2\sqrt{\pi}}\int_0^\infty \frac{dt \exp-(\xi/t - t^2)\, X(-t)}{t\Lambda_+(t)\,\Lambda_-(t)}\right)$$

where $n(\xi)$ is given by (3.1).

The field direction is therefore such that ions going to the electrode are accelerated by the field. As $\xi \to 0$, the derivative $dn/d\xi$, and hence E also, diverges logarithmically, i.e., the conditions for quasineutrality are not satisfied close to the electrode. This means that, when $\xi \sim a/v_0\tau$ (a is the Debye radius) (i.e., within the Langmuir layer), Poisson's equation must be used explicity.

LITERATURE CITED

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